Towards Convention Propagation In Multi-Layer Social Networks

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Abstract. Most work on convention propagation have focused on single-layer networks. This work a) highlights the need for studying convention propagation in multi-layer networks, b) presents results comparing the speed of convention propagation in single-layer vs. multi-layer networks and c) demonstrates the role of influencer agents in convention propagation in multi-layer networks.

1 Introduction

The impact of different configurations of networks on the spread of information, rumours, norms, diseases etc. is being studied in many disciplines including sociology and epidemiology. Several other domains such as traffic management and marketing have benefited from the use of network propagation principles to their advantage. For example, the knowledge of network principles have been used in re-routing passengers arriving in a central hub that provides different modes of transport [5]. Marketing personnel could identify paths in networks that can expedite the faster uptake of a particular product [1, 2]. In this paper, we are interested in the spread of conventions as studied in the field of normative multi-agent systems (NorMAS) using simulations [10].

A simple view of networks categorizes networks into single-layer and multi-layer networks. A multi-layer network consists of many layers where a node (or nodes) in one of the layers might also be present in the other layers. There have been a wide-range of studies on the characteristics of different types of single-layer networks such as random, small-world and scale-free networks (see [7] for an overview). Researchers in NorMAS have investigated how conventions propagate in single-layer networks [3, 11]. However, the characteristics of information diffusion in multi-layer networks has attracted researchers' attention only recently [4, 8] in various domains. In MAS, the interest in exploring the applicability of multi-layer networks has been very recent [6] and this paper investigates the spread of conventions on top of multi-layer networks.

2 Single vs. multi-layer networks

This section aims to present an overview of single vs. multi-layer networks to highlight the need for studying convention propagation in multi-layer networks.

A single-layer network normally refers to individuals connected in a particular context such as work and hobby. Nowadays, it is estimated that an average individual is a member of more than one social network. For example, an individual may be a part of LinkedIn, Facebook and Google+ networks. Each network can be considered as a layer. So, an individual who has an account in all the three networks is a part of a three layer network. However, the nodes an individual is connected to in a particular layer will be different. An individual in a multi-layer network can spread the information from one network to other networks.

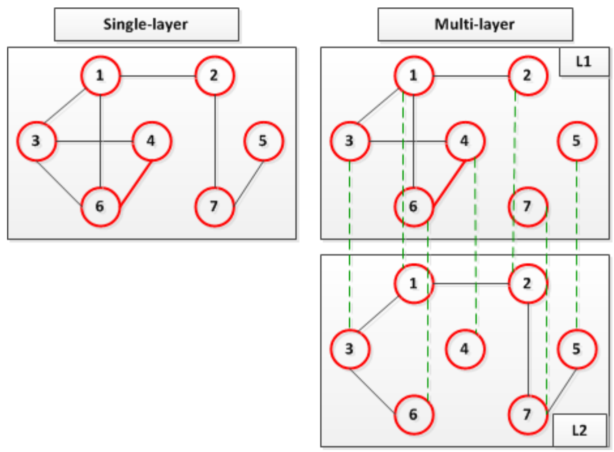

Fig. 1. An example of single and multi-layer networks

Figure 1 shows an example of single- and multi-layer networks. The single-layer network on the left shows the structure of a community where individuals are modelled as nodes and their relationships are shown as links (or edges). The multi-layer network on the right shows the different layers of relationships between individuals in a community. The first layer L1 could correspond to a social network between members on Facebook and the second layer L2 could be the Word of Mouth (WoM) network in the community. The dotted lines indicate that the nodes in L1 and L2 are the same individuals. It can be observed that not all nodes are connected to others in the same layer. For example nodes 5 and 7 are connected in L2, but not in L1. It can be observed that L1 and L2 can be superimposed to create the single-layer network on the left. Superimposition here implies the graph sum[1] of L1 and L2. If L2 is superimposed on top of L1, we get the same nodes (all seven) and the eight links (all links in L1 and L2 with the elimination of duplicates). The superimposition of L1 on top of L2 will yield the same result. So, the order of superimposition does not matter.

Most work that consider network topologies have restricted themselves to a single-layer network where the differences in influences of nodes that are spread across different layers have not been investigated. For example, nodes 3 and 4 are only connected

[1] A brief description of graph sum can be found at http://mathworld.wolfram.com/GraphSum.html

in L1 and not in L2. These two nodes represent individuals that are further apart (physically) but are connected through an online social network. Such differences are often ignored in single-layer networks that aggregate different layers into one through superimposition of layers, since the aim of these networks is to provide a simplified view of relationships in a community. However, differentiating different layers of networks is important because people are connected to each other through a range of networks and the *layered* structure has a significant impact on the dynamics of information propagation in a community of agents such as the speed of convention propagation and the extent of spread in a network.

In what follows, we compare the speed of information propagation of single and multi-layer networks. The speed of information propagation between two nodes is referred to as cascade time (CT) in a network and is measured in hop lengths. Using the simple example shown in Figure 1, we compare the CTs in the two types of networks. Table 1 shows the CTs between two nodes in five different network settings. Assuming that the time taken for information propagation between two adjacent nodes that are connected is one, the CT for the path from nodes 4 to 5 in single-layer network (SL) is 5. The information propagation from nodes 4 to 5 is unachievable in L1 or L2 alone because there is no path between the two.

Each layer in a multi-layer network has certain properties. *Medium delay* (D_m) represents the time taken for a piece of information to flow between two adjacent nodes in a given layer. For example, the D_m in WoM networks is generally greater than online networks such as Facebook or Twitter. When information propagates across layers from the same node, there would be a delay because of *medium switch* (D_s). For example, D_s captures the time to transfer a message from Twitter to Facebook or from a WoM network to Twitter. The CT between nodes a and b is given by:

$$CT_{ab} = \sum_{i=1}^{x} D_{m(L1)} + \sum_{j=1}^{y} D_{m(L2)} + \sum_{k=1}^{z} D_s \tag{1}$$

where *x* represents the number of links in layer 1 through which information propagates, *y* represents the number of links in layer 2, and *z* represents the node switches between the two layers.

Using the formula presented above, we compute the CT along the path from node 4 to 5 in the multi-layer network shown in Figure 1 under three varying circumstances.

Case 1: ML_0 represents a multi-layer network where D_m=1 between two adjacent nodes in both layers (L1 and L2) and there is no medium switch delay (D_s=0). Let us assume that the information originates in node 4 in L1. In order for this information to be spread to node 5, this information needs to cross layers (i.e. L2 to L1 at node 2). The path followed in L1 would be 4-3-1-2. So, the sum of D_m along the path from 4 to 2 in L1 is 3. The information needs to cross layers since a path exists in L2 between nodes 2 and 5. The path followed in L2 is 2-7-5. So, the sum of D_m along the path from nodes 2 to 5 in L2 is 2. The sums of D_m for the path from nodes 4 to 5 is 5. Since D_s is zero, CT_{45} is 5.

Case 2: ML_1 represents a multi-layer network where D_m=1 between two adjacent nodes in both layers L1 and L2 and the medium switch delay is one (D_s=1). In this

Cascade time	SL	L1	L2	ML_0	ML_1	ML_2
CT_{45}	5	-	-	5	6	8
CT_{67}	3	-	4	3	4	5

Table 1. Cascade times along two different paths in five different network configurations

case CT between nodes 4 and 5 becomes 6. CT for ML_1 is one unit higher than ML_0 because of the network switch at node 2.

Case 3: ML_2 represents a multi-layer network where D_m=1 in L1 and D_m=2 in L2 and medium switch cost (D_s=1). This models a case where medium delays are different in different layers (e.g. the medium delay cost in Twitter might be 1 and for WoM might be 2). In this case, the CT between 4 to 5 rises to 8. The D_m along the path between 4 to 2 in L1 is 3. The D_m along the path between nodes 2 and 5 in L2 is 4. The medium switch delay at node 2 is 1. So, CT_{45} in this case is 8.

The same type of calculations for information propagation between nodes 6 and 7 (where origin is node 6 of L2) are presented in the second row of the table. The three cases presented above demonstrate that multi-layer networks are at best equal to single-layer networks (case 1) and most often are slower than single-layer networks (cases 2 and 3). Multi-layer networks are often slow in information propagation because the spread is influenced by the nature of the underlying medium and/or the network switch delays across layers.

3 Experimental model

In this section, we discuss how to construct different types of network topologies and also model how conventions propagate on top of these networks.

We employ a simple contagion spreading model for the study of convention propagation. Similar to the spreading of a contagion such as a virus, a convention propagates from one node to another if one of them has the convention and there is an edge connecting the two. For example, if node A is connected to five other nodes, then, upon adopting a convention, node A spreads the convention to other agents. In the next time step, all the five nodes adopt the convention. This simple model has been used so as to focus on the dynamics of convention propagation in different types of networks. A similar contagion-based model has been used to study convention spreading [9].

While there have been multitude of models for generating single-layer networks (see [7]), there are relatively few approaches used in the literature to model multi-layer networks. Two approaches used are the merging and the splitting approaches where a multi-layer network is built by either merging two simple networks or splitting a simple network into multiple networks [6]. We employed the splitting approach in this work.

A splitting approach for the generation of multi-layer networks - We created an initial single-layer network consisting of certain number of agents (e.g. 100). Three types of networks were considered - Erdos-Renyi (ER) random network, Watts-Strogatz (WS) small-world network and Barabasi-Albert (BA) scale-free network. Once a single-layer network is generated, in order to create the multi-layer network consisting of two-layers, we duplicated all the nodes in the single layer in both layers of the multi-layer network. Then using the two splitting mechanisms (type 1 and 2) discussed below we assign the links to the two-layers.

Type 1 - For type 1, edges from the single-layer network are split between the two layers of the multi-layer network using a pair of probability values. For example, let us consider values 0.1 and 0.9. For each edge in the single-layer network a random number between 0 and 1 is generated. If the random number (r) is less than 0.1, a new edge is created in layer 1. Else if r is between 0.1 and 0.9 a new edge is created in layer 2. Else, the edge goes in layer 1 or layer 2 based on generating a new probability value between 0 and 1 (i.e. if the new value is less than 0.5 the link goes to layer 1 or else it goes to layer 2). Using this approach, an edge is assigned only to one of the two layers. This splitting mechanism generates interdependent networks. Nodes in these type of multi-layer networks remain in a single layer but send information back and forth to other layers.

Type 2 - These multi-layer networks differ from type 1 in that they allow duplication of edges between the two layers (i.e., the same link between any two nodes of the single-layer network can exist in different layers). The else part of the condition in Type 1 is modified. If the random number r is greater than or equal to 0.9, then the edge is added to both layers. So, there is a 10% chance that each edge is present in both the layers. Type 2 models modern social networks where the same individual in a community can belong to two different networks and can be connected to the same member(s) of the community in both of these networks.

To study the spread of conventions, we choose a certain number of individuals who are the originators of conventions (or norm entrepreneurs). In the experiments we conducted, we measured the time taken for a convention to spread from this originator to the entire network.

4 Experiments

In this section we describe the experiments conducted to compare the speed of convention propagation in two different networks and the role of influencer agents on convention propagation.

4.1 Convention propagation speeds in single- vs. multi-layer networks

Section 2 shows that the cascade times along certain paths are slower in multi-layer networks than single-layer networks. In this experiment we empirically investigate the average cascade times for convention convergence in the whole network. For the single-layer network, we randomly chose an agent as the norm entrepreneur which starts propagating the convention. We first generated a particular type of network (e.g. Erdos-Renyi (ER) network) with 100 nodes. Then for each network, we conducted 10 experiments. In each experiment a randomly selected node becomes the norm entrepreneur. We conducted experiments on 100 networks of the same type (i.e. ER network). In total 1000 experiments were conducted. Also, we conducted similar experiments for small-world and scale-free networks. We measured the average time taken to reach a 100% convergence.

Multi-layer networks were also initialized using a similar approach. Convergence times were measured for multi-layer networks that were generated using Type 1 and Type 2 mechanisms. We investigated Types 1 and 2 using three different pairs of probability values for edge assignment. These were a) 0.1 and 0.9, b) 0.3 and 0.7 and c) 0.5 and 0.5. The experiments for multi-layer networks for different probability value pairs are named Types 1a, 1b, 1c, 2a, 2b and 2c (6 experiments).

The experimental results for a Watts-Strogatz (WS) small-world network is shown in Figure 2. Two observations can be made from the figure. First, convention propagation in SL is always faster than ML network. This is because of the delay in network switches which are absent in SL network. Second, the convention propagation in type 2 is faster than type 1 setting. This is because of the duplicate links in type 2 which increase the speed of convention propagation in type 2 multi-layer network. However, the smaller proportion of these duplicate links in the multi-layer type 2 network isn't sufficient to match the speed of convergence to that of the SL network. These were also noted in ER and BA networks. These fairly intuitive results confirm that multi-layer networks indeed slow down convention spreading in realistic settings.

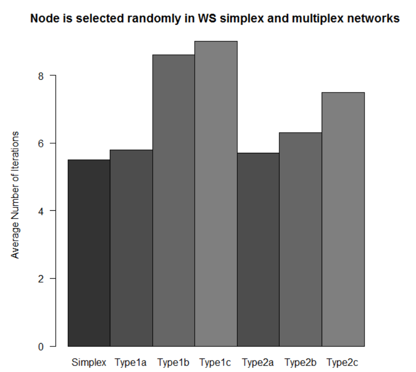

Fig. 2. Convention propagation in WS networks when node is selected at random

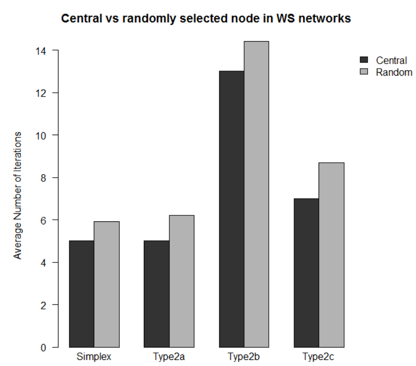

Fig. 3. Comparison of convention propagation using central node vs random node in single-layer and multi-layer networks

4.2 Role of centrality-based influencer agents

To investigate the role of influencer agents, we conducted experiments that compare the speed of convention propagation in both of these networks by starting convention propagation by selecting a different agent to start convention propagation. In the first approach, a norm entrepreneur agent was chosen randomly. In the other approach, based on degree centrality, a hub node is chosen. This hub node has an account in both layers. We conducted 100 experiments (each experiment with a different network of a certain type such as ER network).

The result of this experiment is shown in Figure 3. It can be observed that starting convention propagation using a node selected based on degree centrality is faster than using a node selected at random. This is consistent with the definition of centrality that the centrality of a node measures a node's relative importance within a network. The more central the node, the more influential it is, in the network.

4.3 Role of multiple influencer agents

As opposed to the use of single influencer agent as discussed above, we conducted experiments using multiple influencer agents. The influencer agents in a network are measured by ranking nodes in descending order of degree centrality. The impact of using multiple influencer agents in an ER single-layer network is shown in Figure 4. It can be observed that, even though the hub node is the most connected in any given network, using this node in isolation is slower than using multiple agents to start convention propagation. It can also be observed that as the number of influencer agents increases, the number of iterations taken to fully converge decreases. This is because propagation is started from multiple sources that are located in different areas of the network. This allows the information to be spread over a longer distance much faster than if only one influencer agent is used. Figure 5 shows the impact of using multiple influencer agents in an ER multi-layer network. It can be observed that as the number of influencer agents increases, the speed of propagation also increases. This is consistent with the results for the ER single-layer network. We note that the role of influencer agents in single-layer networks have been studied by other researchers [3,11]. Our study shows that the results hold for multi-layer networks also.

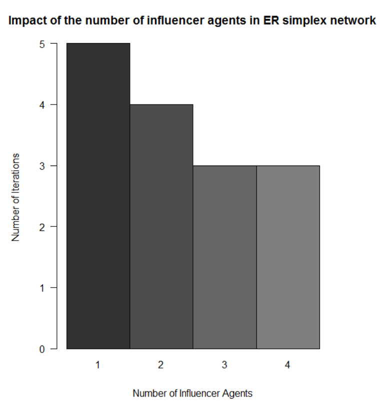

Fig. 4. Convention propagation using multiple influencer agents in an ER single-layer network

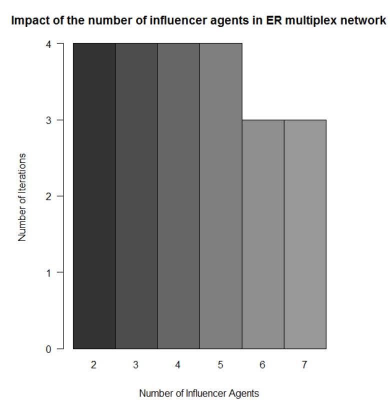

Fig. 5. Convention propagation using multiple influencer agents in an ER multi-layer network

4.4 Discussion

Our work is different from other researchers in the area of NorMAS (e.g. [3, 11]) as they are limited to single layer networks. The work of Li et al. [6] considers multi-layer networks, but assumes that the same nodes are present in the all the layers of the network. In our work, we relax this assumption and allow for nodes to be absent from one of the layers of the multi-layer network.

This work in progress paper makes three contributions to the study of convention propagation in multi-layer networks. First, we have shown that the spread of information is faster in single-layer networks in most realistic situations than multi-layer networks. Second, we have shown through our experimental results how the spread of convention propagation can be improved in single-layer and multi-layer networks through the use of seeded influencer agents. We are currently extending our model by considering the weights of edges since some individuals are more influential over others. In the future, we will investigate a threshold based model where an individual adopts a convention if certain proportion of its neighbours adopt it. Also, the agent will have the ability to decide whether to adopt a convention and whether to propagate a convention to its neighbours.

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