Analysing the Dynamics of Norm Evolution using Interval Type-2 Fuzzy Sets

Christopher Frantz*, Martin K. Purvis*, Mariusz Nowostawski† and Bastin Tony Roy Savarimuthu*
* Department of Information Science, University of Otago, New Zealand
Email: {christopher.frantz,martin.purvis,tony.savarimuthu}@otago.ac.nz
†Faculty of Computer Science and Media Technology, Gjøvik University College, Norway
Email: mariusz.nowostawski@hig.no

***Abstract*—Building on our previous work, we examine the effect of role specialisation on a society using a scenario from the area of comparative economics. In connection with this our central contribution is the use of Interval Type-2 Fuzzy Sets to model emerging normative understanding, which is represented using the concept of Dynamic Deontics. This integrated mechanism, along with a supporting visualisation component, permits a refined inspection of normative alignment on individual, group and society levels beyond mere quantitative analysis, thus offering a powerful extension to the institutional modeller's toolbox.**

Keywords*—*Institutional Modelling, Dynamic Deontics, Fuzzy Sets, Interval Type-2 Fuzzy Sets, Maghribi Traders Coalition, Genoese Traders, Norm Emergence, Norm Evolution

I. INTRODUCTION

The area of institutional modelling and analysis has gained increasing attention in the social sciences, and the field of economics in particular [1, 11], with the intent to identify the institutional environments that make some societies fare better than others, despite otherwise comparable environmental conditions. In this paper we introduce and employ new agent-based social modelling techniques to enhance the state of the art in institutional analysis. For the purposes of this work, we invoke North's interpretation of institutions as "humanly devised constraints that structure [...] social interaction [and] consist of both informal constraints and formal rules." [17]

The advent of formal institutions, such as the laws and regulations, are generally considered to have been crucial to the prosperous development of European societies. Early examples of such institutional frameworks include the *lex mercatoria* and the *Champagne fairs* [15]. An important and often neglected aspect to the development of formal institutions is the presence of *informal* counterparts, such as norms, which may precede laws, or at least be in harmony to foster fruitful social development. However, the emergence of norms – in contrast to the emergence of formal institutions – remains mysterious [16], and the concept has been argued from different perspectives. Those viewpoints (see e.g. Greif [11]) include advocates of the intentional design stance (driven by individuals to further their own or collective interests, e.g. Williamson [25]) in opposition to supporters of the emergence stance (for whom institutions arise without prior conceptualisation).

In our previous work we have concentrated on the latter stance, by conceiving norms as deriving from social interactions, and integrating the notion of initial opportunistic learning with norm internalisation and enforcement, which we represented using the notion of Dynamic Deontics [7]. We introduce this concept briefly in Section II. To demonstrate how Dynamic Deontics can be used to analyse a society's normative landscape, we employ a scenario from the area of comparative economics to explore the effect of role specialisation (Section III), an accepted signature aspect of 'modern' societies for both Smith [21] and Durkheim [5]. To expand analytical capabilities to achieve an integrated perspective on diverse normative understandings, we use fuzzy sets in combination with density-based clustering, the former of which are described in Section IV. Section V summarizes the contributions, and offers an outlook on future work.

II. DYNAMIC DEONTICS

The concept of Dynamic Deontics to represent emergence and change in norms is motivated by the limitations to explain norm emergence and evolution based on conventional deontic logic [23], that commonly relies on primitives representing obligations (e.g. *must*), permissions (e.g. *may*) and prohibitions (e.g. *must not*). Conventional deontic logic has been shown to provide representations that are well understood for the purpose of practical reasoning by human subjects [2]. However, this approach uses a static perspective on norms and does not explain how norms can evolve in a bottom-up fashion or shift as a reaction to the introduction of laws or changing social patterns (see e.g. [4]). Consider for example how norms related to child spanking, public smoking, and the use of language in public media have shifted in the past decades. A further limitation with the conventional norm characterization is that it reduces everything outside a prohibition or obligation to a mere permission. We believe that a continuous representation of norms allows a more truthful and comprehensive representation of a society's norms. The concept of Dynamic Deontics, schematically shown in Figure 1, does so by modelling three essential characteristics:[1]

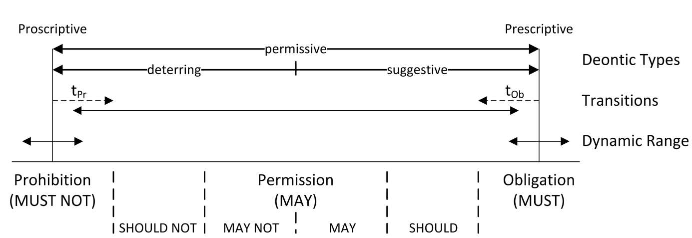


Fig. 1: Conceptual Overview of Dynamic Deontics

[1] Previous work [7] covers motivation and concept in greater depth.

This is a preprint copy of the following publication:
C. Frantz, M. K. Purvis, M. Nowostawski, B. T. R. Savarimuthu: Analysing the Dynamics of Norm Evolution Using Interval Type-2 Fuzzy Sets. IEEE/WIC/ACM International Joint Conference on Web Intelligence (WI) and Intelligent Agent Technologies (IAT), Warzaw, vol. 3, pp. 230-237, 2014, http://ieeexplore.ieee.org/xpl/articleDetails.jsp?reload=true&arnumber=6928190

a) Continuous Notion of Deontics: Norms are not exclusively prescriptive, proscriptive or permissive, but bear additional nuances to represent suggestive (e.g. *should*) or deterring (e.g. *should not*) aspects (see Figure 1). As motivated above, this permits a more refined representation.

b) Stability: The core function of institutions, and thus norms, is typically associated with minimizing uncertainty (see e.g. North [16]). Therefore, stability is an important characteristic of institutions. Although this feature is implicit in the traditional trifurcation of deontics, it is challenged by the potential continuous movement along the deontic scale. To accommodate this aspect, our concept includes stability thresholds near the extremes of prohibitions and obligations (denoted as t_{Pr} and t_{Ob} respectively). If norm understandings dwell in these areas for some specified time, the extremal deontic notions become entrenched and *stick*. Only prolonged deviation from those extremes can result in a shift to more permissive notions.

c) Dynamic Deontic Range: To model both the effect of diverse individual backgrounds (e.g. based on individual experience, culture, socio-economic differences, etc.) and individuals' changes over time, we consider the deontic range (i.e. that demarking the range from greatest pain to greatest gain) to vary across different individuals. This involves both including the experiential range of an individual as well as the allocation of an individual's range along an imaginary absolute global deontic continuum. In addition the width of the deontic range can change over time.

III. Role Specialisation vs. Role Unification in a Historical Trader Scenario

A. Background

One of the motivations for this work is the limited usefulness of norms as governing institutions for larger open societies in which individuals develop specialised skillsets that are constitutive for economic development. The importance of such specialisation is reflected in Durkheim's concept of organic solidarity, and has implicitly been considered by Acemoglu and Robinson [1] with respect to the power structures that promote or inhibit the autonomy of individuals to engage in economic interactions. In this context Purvis and Purvis [18] proposed the CKSW meta-model that is based on Acemoglu and Robinson's analysis and reflects the role of skilled workers as a fundamental driver of a society's economic development.

The example instantiated here builds on a previous model by Frantz et al. [9] that explores this effect of role specialisation using a historical scenario from the field of comparative economics, the *Maghribi Traders Coalition* [11]. This tightly-knit medieval North African traders collective specialised in long-distance trading but managed to operate using *informal* (i.e. not legally-backed) institutions, such as norms, that stipulated proactive denunciation of cheaters as a means of maintaining reputation. A central but relatively unexplored characteristic is the fact that those traders largely performed similar trade-related tasks [10, 11], including the acceptance and handling the goods of fellow traders, engaging in overseas trade themselves, but also delegating trade activities to their associates in a reciprocal fashion. Thus their roles were inclusive with respect to all the key trading practices.

In contrast, the institutional arrangements in Genoese and Venetian trader societies of that period had some significant differences. Both of those Italian trader societies inherently relied on role *specialisation*; traders preferred being investors who concentrated on the allocation of funds to promising trade ventures without necessarily having any tractable experience in performing agent services in long-distance trade themselves. In fact the laborious part of the trade relationship was offered to individuals that often saw it as a temporary, one-off, job opportunity to make money without committing to any monetary investment on their part. Agency relations of that nature were largely regulated using *formal institutional instruments* such as the 'commenda'[2], that offered a contractual framework that allowed one partner to invest funds (the 'commendator') while his counterpart (the 'tractator') facilitated the actual operation. Contracts of this nature could then be enforced in commercial courts. However, the central aspect of interest in this context is the role stratification of the Italian trader societies, leading to the suggestion that the diverging interests of involved parties (investors, merchants) could have been at least a partial driver to install formal institutional mechanisms in order to maintain compliant behaviour.

Essentially, the Maghribi Trader Coalition features role integration, while the Genoese and Venetian traders featured role specialisation. In the following section we use this example to retrace our model foundations, and introduce the analytical extensions that represent the core contribution of this work.

B. Model

The commission-based trading model follows the metaphor outlined above and comprises investors (abbreviated as 'Inv'), who can send goods to randomly chosen 'merchant' trading partners (abbreviated as 'Mer'), who can perform trade in a compliant manner (return the full profit), or cheat (withhold some profit). The investor, in turn, can react to the merchant's behaviour. To represent the effect of actions and reactions, parties involved in an action receive payoffs shown in Table I.

Action-Reaction combinations		**Utility from actions**	
Action (Mer)	**Reaction (Inv)**	**for Mer**	**for Inv**
TRADE FAIR	FIRE	-2	-1
TRADE FAIR	RETALIATE FAMILY	-3	-1
TRADE FAIR	PAY COMMISSION	1	1
WITHHOLD PROFIT	FIRE	-1	0
WITHHOLD PROFIT	RETALIATE FAMILY	-3	1
WITHHOLD PROFIT	PAY COMMISSION	2	-2

TABLE I: Action Reaction Feedback Combinations

For example, if a merchant acts compliantly and trades fair but is nevertheless fired (Row 1), it receives negative feedback (-2), while the investor who fires a compliant trader despite honest trading potentially loses all future returns from a compliant trader, and thus receives a negative feedback (-1). In contrast, withholding profit followed by a retaliation against the merchant's family (Row 5) will have very negative impact on the merchant (-3), while being associated with a degree of satisfaction for the betrayed investor (1). Agents memorize experiences made for particular actions and use those to learn about the action choice that renders them with the highest reward. This is further discussed in the following section.

[2]Refer to van Doosselaere [22] for more detailed information on the structure of commenda relationships in the medieval Genoese society.

In addition, agents can act as third-party norm enforcers for observed behaviour by imposing their own reactions based on their own experience. Note that the third-party enforcement is likewise internalised by individual agents, and thus, in combination with their individual experiences, drives their behaviour. For algorithmic details of the scenario refer to [9].

C. Operationalising Dynamic Deontics

For our central objective of interest, the analysis of a differentiated normative understanding based on experiential learning, we operationalise the concept of dynamic deontics using reinforcement learning (Q-Learning) [24], as described in [7]. Individuals can build their individualised understanding of the institutional environment, which, in this case, is based on the actions they explore. Using Q-Learning, the deontic range is delimited by the respective highest and lowest Q-values in the agent's situational memory, thus representing an agent's greatest gain and greatest pain. Based on new experience this range can expand. Q-Learning's memory discount characteristics, in contrast, represent the agent's forgetfulness (as a representation of bounded rationality [20]), and thereby assures the dynamic nature of the deontic range.

To provide some semantic reference to the numeric representation of the deontic range, we assign labels to different compartments along the deontic range, extending from *must not* (to express prohibitions) via *should not* to *may not*, to *may*, *should* and *must* (see Figure 1). Note that *may* and *may not* are two labels we use to signify a permission that is either somewhat encouraged or somewhat discouraged, respectively. All those labels are assigned to equally-sized *deontic compartments* that are symmetrically mapped to the deontic scale.[3]

Over time individuals develop an understanding about particular actions and their effects. To derive the agent's normative understanding of a certain action, such as TRADE FAIR, all Q-Learning entries (which are action-reaction pairs such as [TRADE FAIR, PAY COMMISSION]) are grouped by the respective action (in this example TRADE FAIR). Given the existence of a variety of action-reaction pairs, their respective values need to be aggregated to derive a deontic value the action can then be associated with on the deontic scale, a process we will exemplify in the following. We base this aggregation process on the assumption that individuals focus not on the mean of possible consequences, but rather on the most extremal values (e.g. a sanction that has the strongest negative reinforcement, i.e. is feared most).

Showing a situational extract from an individual agent in the nADICO syntax [8] (Figure 2)[4], helps to visualize this process. In nADICO the consequences of actions are represented as nested so-called institutional statements (Level 1 in Figure 2), to construct a normative statement for the action on Level 0 (here 'WITHHOLD PROFIT'). To aggregate individual action-reaction pairs and to determine the overall value for the leading action (deontic value), the Q-values of all consequences of a given action are first summarized to determine what we call the *deontic bias*, i.e. whether the overall understanding for that action is biased towards the deterring or suggestive side of the deontic range. In the example below, this is *deterring*, inasmuch as (-19.26 + 5.38 - 1.42) $<$ -4.98 (-4.98 is the center of the deontic range, calculated as the midpoint between highest and lowest Q-value). As a second step, the extremal value pointing to the deontic bias is mapped onto the deontic range (with symmetric compartments for deontic terms along that range, as schematically visualised in Figure 1) in order to represent the individual's understanding of what it is supposed (or not supposed) to do (here resolving to -19.26). So for this example the situational deontic range resolves to *should not*. The formalised representation is discussed in [7].

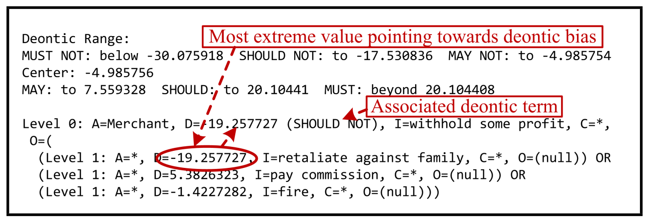


Fig. 2: Example for Deriving Deontic Terms from an Agent's Situational Deontic Range

The remaining aspect – stability – is determined by storing the number of rounds deontic values of a given action resolve to the extreme deontic primitives *must* and *must not*.

For both simulation scenarios, the role-integrated Maghribi society and the specialised Genoese society, the Q-Learning memory instance internalises both feedback from individual learning as well as from norm enforcement, only differentiated by the potential feedback they can experience. Traders in our model of the Maghribi society can expect feedback from both an investor and merchant perspective (see Table I); whereas Genoese traders only experience feedback in their predefined role as either investor or merchant (i.e. either taking the side as acting or reacting party).

D. Simulation

We parameterize the model with settings shown in Table II.

Parameter	Value
Number of agents	100 (Maghribi) / 200 (Genoese)[a]
Tolerance zone around extreme deontics (t_{Pr}, t_{Ob})	0.05 of deontic range amplitude
Norm establishment threshold	100 rounds
Norm destruction threshold	200 rounds
Deontic range history length	100 rounds
Memory discount factor	0.99
Exploration probability	0.1

[a] Genoese numbers are doubled to assure an equal number of merchants in both scenarios, given the Genoese' role specialisation (see [9] for details).

TABLE II: Simulation parameters

We show the simulation results as a time-series (see Figure 3; refer to [9] for more detailed discussion) displaying the fraction of agents whose deontic values resolve to particular deontic terms, e.g. the fraction of agents that believe they *should* withhold profit, etc. In the results shown in Figure 3, we can observe that the Maghribi society shows a continuous convergence towards an understanding that they *may not* withhold profit, along with a stable fraction believing

[3] Note that a more refined allocation along with choice of terms is an aspect of future research.

[4] Besides the individual's norm understanding, at its top the extract shows a numeric representation of the situational deontic range, along with its center point and the boundaries of the derived deontic compartments.

they *must not* withhold profit. For the Genoese merchants it was similarly observed an increasing understanding that they *should* withhold profit to further their own ends. Thus we can observe behavioural convergence of societies over time.[5]

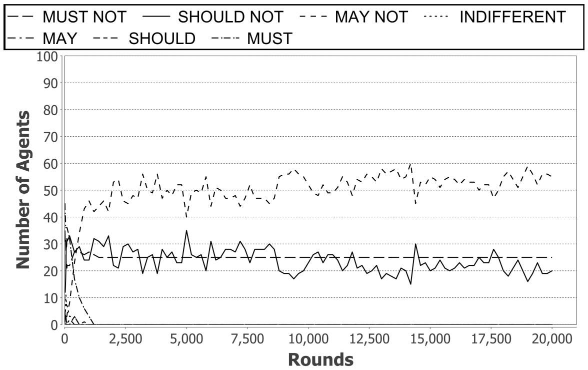


Fig. 3: Evolving Normative Understanding for 'withholding profit' in Maghribi Traders Scenario

Visualizing the results of time-series bears the obvious advantage of retracing the society's changing normative understanding over time. However, it has limitations:

- Micro-level deontic ranges – Individuals maintain an individualised normative understanding, i.e. an individualised understanding of what *should* means etc. Based on mere chart observation we can thus not make any inference about how individuals understand a particular deontic term, i.e. how they allocate it along the deontic scale, both including the *positioning* and *width of deontic compartments* along their individual deontic range.
- Macro-level deontic ranges – Similarly we are limited in what conclusions we can draw about societies as a whole. The category labels neither indicate *how aligned individuals' understanding is* nor contain information about the *distribution of the normative understanding on society level*.

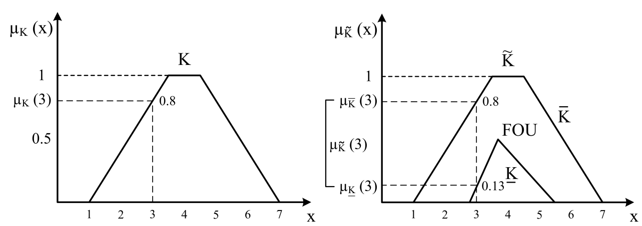
Both aspects are relevant if we want to model societies of different social and cultural make-up, analytically distinguishing collective societies to which we ascribe a more unified norm understanding from individualised societies in which normative understanding could be less aligned.

Statistical analysis is of value as it allows to measure diversity of understanding by indicators such as mean and variance, but it provides limited facilities to integrate different views and more so, to detect ambiguous understandings in an accessible fashion. Unsupervised approaches such as neural networks likewise lack an accessible interpretation, which we deem essential for institutional modelling and analysis in particular. To incorporate ambiguous understandings on *multiple levels* (individual vs. societal perspective), to provide an *accessible systematic integration of intervals* (here: deontic compartments) and to offer a *representation that ultimately enables reasoning about uncertainty* (the latter of which maps well on the purpose of institutions), we apply the notion of fuzzy sets. In the following we will introduce fuzzy sets as a mechanism to inspect deontic ranges of individual agents as well as the entire society.

IV. Fuzzy Sets

A. Background

Fuzzy sets which offer a computational means to deal with uncertainty (and thus match the metaphor of institutions) are suited to model the 'spread' of societal beliefs. In the original notion of fuzzy sets conceptualised by Zadeh [26], input values for a given input domain are evaluated against a membership function that resolves to a *degree of membership* for that respective input value with a fuzzy set, and is expressed as a value between 0 and 1. In the example shown in Figure 4a, the degree of membership with the fuzzy set K for the input value 3 ($\mu_K(3)$) is 0.8. A shortcoming of the original concept is the problem that the membership function itself is crisp. Recalling our context, the intent to integrate multiple different understandings of the same concepts, e.g. the understanding of *should* for agent A and B, we cannot make such an assumption. In fact this characteristic has been recognized as a core shortcoming of the original (type-1) fuzzy set concept (see e.g. [13]). Zadeh further proposed type-2 fuzzy sets [27], in which the membership function itself can bear uncertainty so that a membership function is not a singular function but comprised of an upper ($\overline{K}$) and lower ($\underline{K}$) membership function that border what is called the *Footprint of Uncertainty* (FOU), which is exemplified in Figure 4b.



(a) Type-1 Fuzzy Set (T1FS) (b) Type-2 Fuzzy Set (T2FS)

Fig. 4: Type-1 vs. Type-2 Fuzzy Sets

Consequently, degree of membership is described as an interval, such as [0.13, 0.8] describing the degree of membership for $\mu_{\widetilde{K}}(3)$ with fuzzy set $\widetilde{K}$ in the example in Figure 4b.

With respect to our objective to develop an integrated view of individual members' membership functions on arbitrary aggregation levels (e.g. per individual, group, or society), second-order fuzzy sets offer a suitable representation. Particularly appropriate are Interval Type-2 Fuzzy Sets (IT2FS), which afford the representation of both individual compartments and the deontic range as intervals.

B. Applying Fuzzy Sets to the Simulation Scenario

To operationalise IT2FS for the purpose of generalising individuals' subjective understanding of given terms, we employ the methodological approach used by Zadeh [28] and Liu and Mendel [14] for *Computing with Words*. In order to derive an overall membership function (MF) that offers analytical value, individual MFs need to show at least a principal overlap

[5]For a more detailed discussion of the results and their implications refer to [9]. Our focus here lies on improving the means for representing normative understanding.

in order to arrive at a lower MF (i.e. the extent to which individuals are somewhat[6] certain that they have a shared understanding of the concept of interest (expressed as $\underline{K}$ in Figure 4b)). To arrive at this 'basic understanding', agents' individual intervals representing the deontic compartments require statistical preprocessing. To do this, Liu and Mendel [14] used a four-step preprocessing operation prior to the generation of MFs, from which we adopt the latter three steps. Those three steps involve the identification of intervals (a) that appear unreasonably high or low (lying outside 1.5 $\times$ the Interquartile range) (*Outliers*), (b) lying outside a confidence boundary of 95 percent of all intervals (*Outside tolerance*), and (c) that do not sufficiently overlap with other intervals to generate a common lower MF (*Unreasonable data*).[7]

Based on the remaining intervals, MFs for given sets can be generated, which represent the individual deontic terms. So, to perform a real-time representation of the situational understanding, we extend our simulation model with an Interval Type-2 Fuzzy Logic System (IT2FLS) module which we constructed based on documentation and code offered by Liu and Mendel [14]. We further extended this module with visualiser and analyser components. Those enable the observation of the interval preprocessing steps, the MF generation results for individual fuzzy sets (here: individual deontic compartments) while offering further a combined view on all fuzzy sets (here: the entire deontic range). When combined with rule input that associates membership of inputs with one or more fuzzy sets, specific consequences (specified as part of the rules input) can be triggered, e.g. to model graduated sanctioning, an aspect we do not explore in the present work, since our focus lies on the analysis of the emerging deontic ranges.

C. Analysing a Society's Normative Alignment

Figure 5 depicts an example visualisation of the MF generation process for the term *should not* across the entire deontic range, displaying the intervals that have been filtered as part of the preprocessing steps, with the outliers (plotted as dashed dark gray intervals) being the most obvious ones (22), and to lesser extent intervals that lie outside the tolerance boundaries of the remaining intervals (4 intervals marked in orange colour). The remaining 74 intervals have been fuzzified into T1FS (as specified by Liu and Mendel [14]) and used to generate a MF that suggests that for 74 percent of the society *should not* extends at most from -170 to around 30, with higher levels of certainty for values ranging from around -120 to around -10 along the deontic scale. For all following figures we will omit the intervals used for the MF generation as their fuzzification obscures the generated lower MFs (blue colour).

To integrate the individual situational understanding for all deontic compartments, the generated MFs have been integrated to a combined view of all generated MFs along the deontic scale as shown in Figure 6, offering a society view on the prevailing norm understanding. To get an understanding about the representativeness of the generated MFs for the entire society, the labels in the combined view further indicate the fraction of individual intervals that have been included in the generation process for the respective MF.

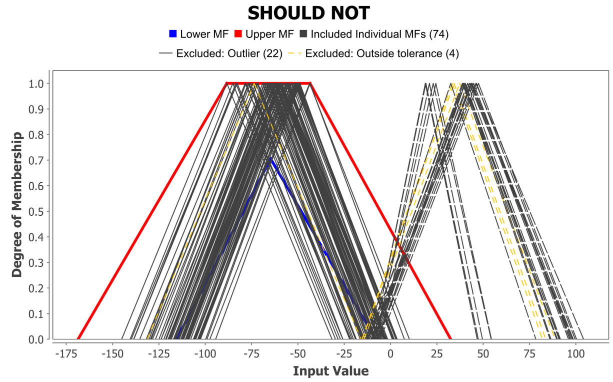


Fig. 5: MF Generation for Deontic Term *should not* in Scenario with Rule Unification

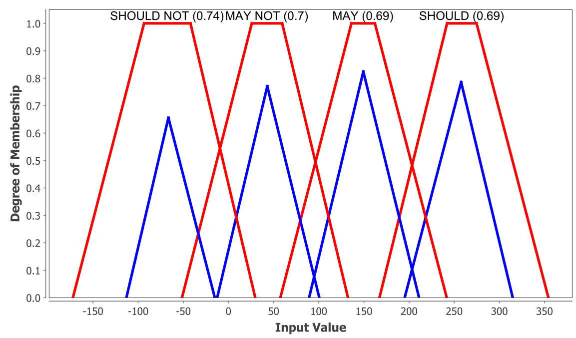


Fig. 6: Generated MFs for the Role Unification Scenario

Returning the interpretation of the simulation results, this integrated view, here representing the Maghribi trader society (in which traders all acted in similar roles), shows the alignment of the normative understanding of the overall society. After around 3000 rounds the generated membership functions integrate at least 69 percent of all individual intervals and do so with a high *lower MF*, i.e. a relatively narrow FOU (and thus relatively low uncertainty), which supports the observation of a strongly aligned normative understanding.

Contrasting this, Figure 7 shows the generated MFs for the modelled Genoese trader society (which had a stratified, i.e. specialised, role understanding). Here we can observe a different picture. Observing the resulting MFs, even after 20,000 iterations the generated MFs offer a weak representation of the agent society. In the best case (deontic term *may*), 20 percent of the individual understandings of *may* can be aggregated into a common interpretation. However, even within these 20 percent we can observe a considerable deviation based on the very low *lower MF* and thus large FOU. In this situational snapshot, only three intervals were used to represent what *should* means, challenging the usefulness of IT2FS to represent collective understanding. Consequently upper and lower MF are nearly identical for this example. For the deontic terms *should not* and *may not* MFs could not be generated, suggesting a too diverse understanding of what those terms entail.

However, to develop a better understanding – note that the integrated view only shows eventually generated MFs – we require a more detailed inspection of the individual generation processes. To facilitate this analysis, we developed a dashboard perspective that integrates the MF generation processes for each deontic term as well as the total view on all generated

[6] The employed methodology [14] fuzzifies T1FS prior to generating T2FS.
[7] These process steps are detailed in [14].

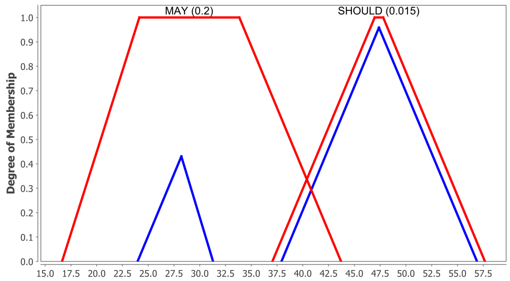


Fig. 7: Generated MFs for the Role Specialisation Scenario

MFs. Figure 8 shows a screenshot of this dashboard for the Genoese society, with individual MFs allocated to the left and the combined view (as seen in Figure 7) on the top right of the dashboard.

Looking at the individual generation processes it becomes apparent that most intervals have been excluded during the preprocessing caused by a tendentially polarised spread across the deontic range in combination with insufficient general overlap of intervals. A particular example is the deontic term *may not*, for which intervals are polarised to either end of the deontic spectrum without any overlapping around the center. In the context of the scenario, the bipolar allocation of intervals based on role specialisation, i.e. individuals making experience from the perspective of their specific role, is retraceable. However, since it is geared for cases in which intervals generally show an overlap and only occasionally deviate strongly, Liu and Mendel's methodology [14] is not able to process such polarised interval spreads, which is retraceable, given their interest to arrive at globally unified MFs.

D. Combining Fuzzy Sets with Density-Based Clustering

Nevertheless, we can observe a clustering of intervals into two to four general clusters. To approximate this clustering, we add an additional preprocessing step based on preclustering of raw intervals before feeding those into the MF generation process. For this we use the density-based clustering algorithm DBSCAN [6], which determines clusters based on the *maximal permissible distance of individual inputs (ϵ)* along with a *minimal number of inputs in close proximity to constitute a cluster*. Here we use a distance measure that is based on four discrete values:

- 0 indicating that intervals have the same midpoint and thus either one interval is the superset or both intervals fully overlap;
- 0.5 indicating that intervals mutually overlap beyond their respective midpoints, i.e. one leg of each interval reaches beyond the other interval's midpoint;
- 1.0 indicating that intervals are overlapping;
- 2 indicating that intervals are not overlapping.

In contrast to the preprocessing done by Liu and Mendel [14], DBSCAN only concentrates on the density of individual intervals but is agnostic about the allocation along the deontic scale. This characteristic bears another benefit that is suitable for the analysis of social scenario. While the IT2FLS generator tries to generalize *one* unified representation for a given term, with guided MF generation based on pre-clustering we can identify subgroups that have a strongly homogeneous understanding of a given concept but show strong heterogeneity with respect to other groups. From a sociological perspective this allows not only the development of a total view on the society (macro-level perspective), but by applying pre-clustering, we can identify how different individuals cluster into groups that develop an incompatible normative understanding of the same concept, i.e. form different clusters for the same concept along the deontic scale. This allows us to explore the meso-level in addition to the macro- (society as a whole) and micro-level (the individual intervals). Guided MF generation can thus in principle be driven by characteristics other than the majority (here: the largest cluster), such as different social choice mechanisms [3], an aspect which depends on the objective of the simulation. However, for the purpose here, we concentrate on the dominating understanding of a society's norms.

Figures 9 and 10 show the results of introducing preclustering with DBSCAN with ϵ (the maximum distance between intervals) set to 0.5 and a minimum of three intervals to constitute a cluster. Looking at the resulting inclusion based on the preclustered intervals given in Figure 10, we can achieve inclusion fractions of more than 40 percent (as opposed to maximal 20 percent without preclustering as shown in Figure 7). The resulting MFs are thus more representative for the understanding of those terms within the modelled specialised Genoese trader society. However, the relatively large FOU (compared to the role-integrated Maghribi model in Figure 6) indicates considerable diversity of views, with increasing levels of alignment for terms *should not*, *may not* and *may*; the agents have a comparatively well-aligned understanding of what *should* means. Extending the observation to the generation processes shown in Figure 10 we can see the effect of the preclustering.[8] For all but one deontic term (*should not*), preclustering resulted in three clusters of which the largest ones have been chosen for the generation of representative MFs.

E. Results Summary

Summarizing the findings with respect to the trader scenario, we can retrace that for integrated roles, individuals develop more aligned experiences (Figure 6). We see this as a supportive factor driving compliant social behaviour. An aligned normative understanding implicitly drives more socially acceptable behaviour, given that trade interactions, both of compliant and non-cooperative nature, can be experienced by either agent and drive a more homogeneous understanding. Our model of the Genoese traders, in contrast, supports the understanding of diverging normative understandings, going as far as to limit the ability to develop an integrated representation for the entire society (Figures 9 and 10). The results allow us to retrace that diverging social behaviour and differing normative understanding can be drivers of social conflict in the process of developing an overall understanding. This supports the hypothesis that socially diverse normative understandings ultimately require explicit effort to consolidate the society's overall make-up, be it by conflict or by establishing formal institutions, such as rules in the form of laws and regulations as a result of social or centralised decision-making processes.

From an institutional modelling perspective, this work contributes an approach that allows the development of an

[8] In Figure 10 detected clusters are differentiated by colours and plotted with half the size of the intervals, with ϵ indicated on the secondary y axis.

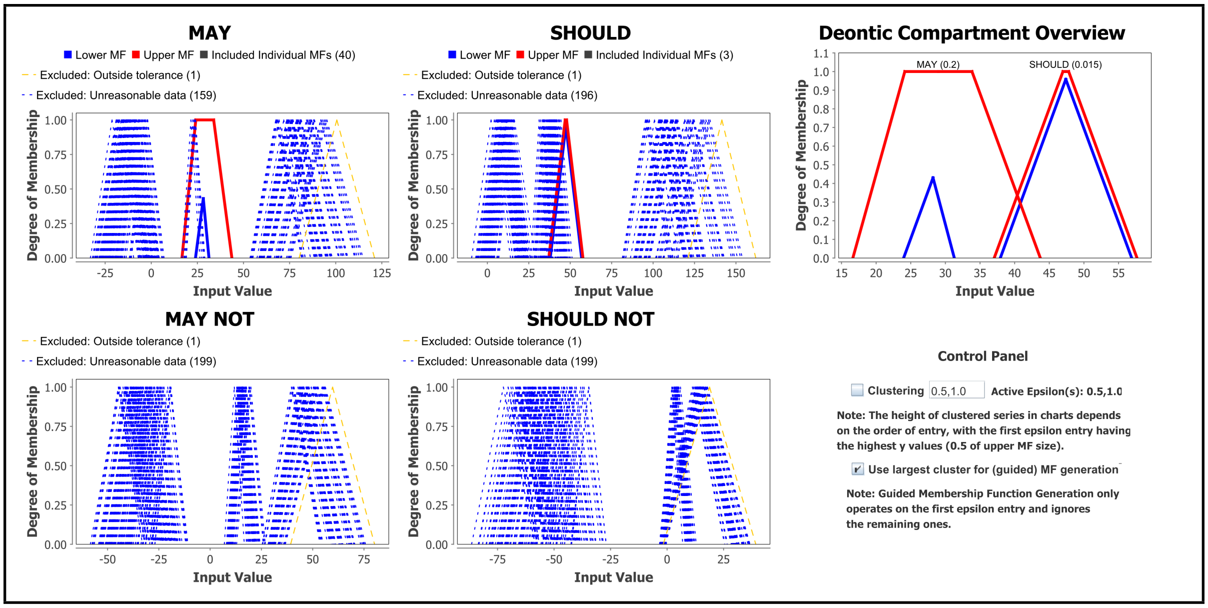


Fig. 8: IT2FLS Dashboard for Role Specialisation Scenario

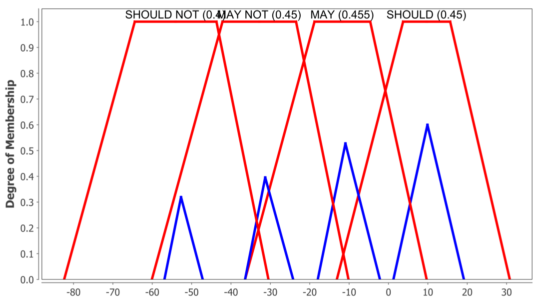


Fig. 9: Guided MF Generation for Role Specialisation

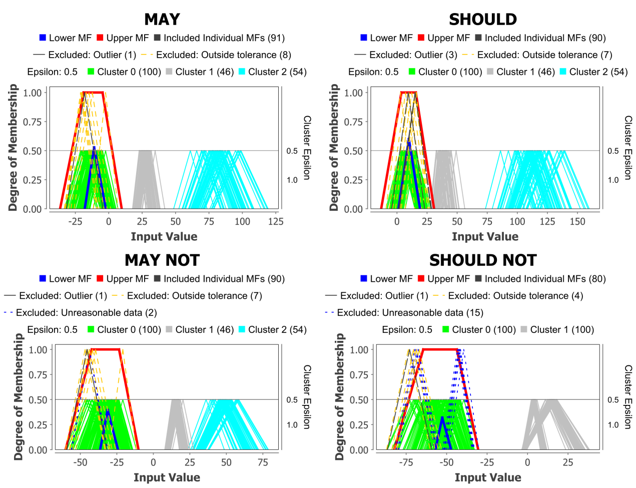


Fig. 10: Dashboard Extract for Guided MF Generation

integrated perspective on individual norm understandings. It utilises the notion interval type-2 fuzzy sets which are particularly well-suited for the representation of imprecise understanding of concepts by individuals and is in line with dynamic deontics' notion of a continuous space lying between the deontic extremes of prohibition and obligation. Important analytical measures for normative alignment include 1) *the fraction of all individuals represented in a MF* (quantity) as well as 2) *the size of the FOU that describes the heterogeneity of the norm understanding* (quality) for individuals considered in the generation process of the respective MF. To improve the inclusiveness of this approach, we further extended it with a preprocessing mechanism based on the density-based clustering algorithm DBSCAN [6] to precluster candidate input intervals in order to drive *guided MF generation*. Besides the purely analytical perspective, there is further value from a sociological and social-psychological perspective. Beyond improving the representativeness of generated MFs with respect to the overall society, the additional preprocessing step allows for the isolated treatment of subclusters of intervals, which represent the societal equivalent of subgroups. This supports the analysis on the meso-level, inspecting entities that exist as constituents of the society as a whole (macro-level), but transcend their individual members (micro-level).

V. Discussion and Related work

The contributions of this work are twofold. We addressed a specific problem in a scenario [11] from the area of comparative economics, which explores why comparable societies have taken different institutional paths to address the important problem of assuring cooperative behaviour. In this context, we focus on the particular characteristic of role integration vs. role specialisation that applies to the otherwise comparable Maghribi and Genoese trader societies, inasmuch as it has been ascribed general importance to discriminate societies in different developmental stages, both from the area of economics [21] as well as sociology [5]. Our simulation supports the hypothesis that role specialisation drives a diversified, if not polarised, normative understanding, whose lacking alignment supports the need for more explicit formal institutions to sustain cooperation. In the Genoese trader society this had been realised by means of formal contractual instruments, such as the commenda, and enforcing institutions, such as commercial courts.

The second, more central, contribution of this work is an approach for representing and analysing normative understanding both on individual and aggregate level by providing concepts and tools for a systematic aggregation. Using fuzzy sets in combination with density-based clustering this can be established on arbitrary social level, and generalised for the use in the wider area of agent-based modelling and social simulation, particularly for scenarios in which, in contrast to many game-theoretical approaches, the uniformity assumption of individuals does not hold.

To date fuzzy concepts have found application in the context of social simulation, such as Hassan et al.'s work modelling friendship dynamics in artificial societies [12]. Outside the area of social simulation fuzzy sets have found wider adoption, including *Computing with Words* [14, 28], as well as in fields outside technical disciplines (e.g. ecology [19]), a detailed discussion of which is beyond the scope of this work. However, as far as we are aware to date no other approaches utilised fuzzy sets (and IT2FS in particular) as a means to model normative understanding on different social levels.

Possible future directions are manifold. Although only used for analytical purposes at this stage, the IT2FLS module implemented as part of this work provides the facilities to utilise established fuzzy sets to realise fuzzy decision-making for individuals. With the established model, we have provided the foundation to let agents actively use the derived normative understandings, such as differentiated sanctioning of violating behaviour. The aggregated normative understanding further provides the prerequisite to model collective action processes such as quorum- or majority-based voting as well as opinion aggregation, be it on group or society level.

We believe that the presented approach – integrating fuzzy shared understanding with institutional modelling – offers promising directions for a more realistic representation of social concepts in the context of agent-based modelling and the field of social simulation in general.

References

[1] D. Acemoglu and J. Robinson. *Why Nations Fail: The Origins of Power, Prosperity, and Poverty*. Crown Business, New York, NY, 2012.

[2] S. Beller. Deontic norms, deontic reasoning, and deontic conditionals. *Thinking & Reasoning*, 14(4):305–341, 2008.

[3] Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. A Short Introduction to Computational Social Choice. In *Proc. 33rd Conference on Current Trends in Theory and Practice of Computer Science*. Springer, 2007.

[4] S. E. Crawford and E. Ostrom. A Grammar of Institutions. In *Understanding Institutional Diversity*, pages 137–174. Princeton University Press, Princeton, 2005.

[5] E. Durkheim. *The Division of Labour in Society*. Free Press (1933), 1893.

[6] M. Ester, H.-P. Kriegel, J. Sander, and X. Xu. A Density-Based Algorithm for Discovering Clusters in Large Spatial Databases with Noise. In E. Simoudis, J. Han, and U. Fayyad, editors, *Second International Conference on Knowledge Discovery and Data Mining*, pages 226–231, Portland, Oregon, 1996. AAAI Press.

[7] C. Frantz, M. K. Purvis, M. Nowostawski, and B. T. R. Savarimuthu. Modelling Institutions using Dynamic Deontics. In *COIN@PRIMA 2013*, 2013.

[8] C. Frantz, M. K. Purvis, M. Nowostawski, and B. T. R. Savarimuthu. nADICO: A Nested Grammar of Institutions. In G. Boella, E. Elkind, B. T. R. Savarimuthu, F. Dignum, and M. K. Purvis, editors, *PRIMA 2013: Principles and Practice of Multi-Agent Systems*, volume 8291 of *LNAI*, pages 429–436, 2013.

[9] C. Frantz, M. K. Purvis, B. T. R. Savarimuthu, and M. Nowostawski. Modelling the Impact of Role Specialisation on Cooperative Behaviour in Historic Trader Scenarios. In *COIN@AAMAS 2014*, 2014.

[10] J. L. Goldberg. *Trade and Institutions in the Medieval Mediterranean: The Geniza Merchants and their Business World*. Cambridge University Press, 2012.

[11] A. Greif. *Institutions and the Path to the Modern Economy*. Cambridge University Press, New York, 2006.

[12] S. Hassan, M. Salgado, and J. Pavón. Friendship Dynamics: Modelling Social Relationships through a Fuzzy Agent-Based Simulation. *Discrete Dynamics in Nature and Society*, 2011, 2011.

[13] G. Klir and T. Folger. *Fuzzy Sets, Uncertainty and Information*. Prentice Hall, Englewood Cliffs, 1988.

[14] F. Liu and J. M. Mendel. Encoding Words Into Interval Type-2 Fuzzy Sets Using an Interval Approach. *IEEE T. Fuzzy Systems*, 16(6):1503–1521, 2008.

[15] P. R. Milgrom, D. C. North, and B. R. Weingast. The Role of Institutions in the Revival of the Trade: The Law Merchant, Private Judges, and the Champagne Fairs. *Economics and Politics*, 2:1954–1985, 1990.

[16] D. C. North. *Institutions, Institutional Change, and Economic Performance*. Cambridge University Press, Cambridge, 1990.

[17] D. C. North. Institutions. *Journal of Economic Perspectives*, 5(1):97–112, 1991.

[18] M. K. Purvis and M. A. Purvis. Institutional Expertise in the Service-dominant Logic: Knowing How and Knowing What. *Journal of Marketing Management*, 28(13–14):1626–1641, 2012.

[19] C. L. Ramírez, O. Castillo, P. Melin, and A. R. Díaz. Simulation of the Bird Age-Structured Population Growth Based on an Interval Type-2 Fuzzy Cellular Structure. *Inf. Sci*, 181(3):519–535, 2011.

[20] H. A. Simon. A Behavioral Model of Rational Choice. *The Quarterly Journal of Economics*, 69:99–118, 1955.

[21] A. Smith. *An Inquiry into the Nature and Causes of the Wealth of Nations*. W. Strahan and T. Cadell, 1776.

[22] Q. van Doosselaere. *Commercial Agreements and Social Dynamics in Medieval Genoa*. Cambridge University Press, 2009.

[23] G. H. von Wright. *Norm and Action: A Logical Enquiry*. Routledge & Kegan Paul, 1963.

[24] C. Watkins. *Learning from Delayed Rewards*. PhD thesis, Cambridge University, 1989.

[25] O. Williamson. *The Economic Institutions of Capitalism*. Free Press, New York, 1975.

[26] L. A. Zadeh. Fuzzy sets. *Information and Computation*, 8:338–353, 1965.

[27] L. A. Zadeh. The Concept of a Linguistic Variable and its Application to Approximate Reasoning - I. *Information Sciences*, 8:199–249, 1975.

[28] L. A. Zadeh. Fuzzy Logic = Computing with Words. *IEEE Transactions on Fuzzy Systems*, 4(2):103–111, 1996.